Lot Tolerance Proportion or Percentage Defective (LTPD). The *lot tolerance proportion defective*, usually denoted by p_t, is the lot quality which is considered to be bad by the consumer. The consumer is not willing to accept lots having proportion defective p_t or greater. 100 p_t is called *Lot Tolerance Percentage Defective*. In other words, this is the quality level which the consumer regards as rejectable and is usually abbreviated as *R.Q.L.* (*Rejecting Quality Level*). A lot of quality p_t stands to be accepted some arbitrary and small fraction of time, usually 10%.

Process Average Fraction Defective **($\bar{p}$).** $\bar{p}$ represents the quality turned out by the manufacturing process over a long period of time. In industry, the quality of any process tends to settle down to some level which may be expected to be more or less the same everyday for a particular machine. If this level could be maintained and if the process is working free from assignable causes of variation, the inspection could often be dispensed with. But in practice, as a result of failure of machine and men, the quality for the product may suddenly deteriorate. The process average of any manufactured product is obtained by finding the percentage of defectives in the product over a fairly long time.

Consumer's Risk. Any sampling scheme would involve certain risk on the part of the consumer—in the sense that he has to accept certain percentage of undesirably bad lots, *i.e.*, lots of quality p_t or greater fraction defective. More precisely, the probability of accepting a lot with fraction defective p_t is termed as consumer's risk and is written, as P_c. Usually it is denoted by β. This is taken by Dodge and Romig as 10% or 0·10.

$$\text{Consumer's risk} = P_c = P\ [\text{accepting a lot of quality } p_t] = \beta \qquad \ldots(1\cdot14)$$

Producer's Risk. The producer has also to face the situation that some good lots will be rejected. He might demand adequate protection against such contingencies happening too frequently just as the consumer can claim reasonable protection against accepting too many bad lots. The probability of rejecting a lot with 100 $\bar{p}$ as the process average percentage defective is called the producer's risk P_p and is usually denoted by α. Thus

$$\text{Producer's risk} = P_p = P\ (\text{of rejecting a lot of quality } \bar{p}) = \alpha \qquad \ldots(1\cdot15)$$

Rectifying Inspection Plans. In the following sections we shall discuss lot by lot sampling plans in which a specified quality objective is attained through corrective inspection of rejected lots. The inspection of the rejected lots and replacing the defective pieces found in the rejected lots by the good ones, eliminates the number of defectives in the lot to a great extent, thus improving the lot quality. These plans are called '*Rectifying Inspection Plans*' and were first introduced by Harold F. Dodge and Harry G. Romig of the Bell Telephone Laboratories before World War II. These plans enable the manufacturer to have an idea about the average quality of the product that is likely to *result* at a given stage of manufacture through the combination of production, sampling inspection and rectification of rejected lots.

Most of the rectifying inspection plans for lot by lot sampling call for 100% inspection of the rejected lots and replacing the defective pieces found by good ones. The two important points related to rectifying inspection plans are :

(*i*) The average quality of the product after sampling and 100% inspection of rejected lots, called Average Outgoing Quality (*AOQ*); and

(*ii*) The average amount of inspection required for the rectifying inspection plan, called Average Total Inspection (*ATI*).

TABLE : FACTORS USEFUL IN THE CONSTRUCTION OF CONTROL CHARTS

	Mean chart			Standard deviation chart					Range chart				
Sample size	Factors for control limits			Factors for central line	Standard deviation chart				Factors for central line	Range chart			
n	A	A_1	A_2	c_2	B_1	B_2	B_3	B_4	d_2	D_1	D_2	D_3	D_4
2	2.121	3.760	1.886	0.5642	0	1.843	0	3.297	1.128	0	3.686	0	3.267
3	1.232	2.394	1.023	0.7236	0	1.858	0	2.568	1.693	0	4.358	0	2.575
4	1.500	1.880	0.729	0.7979	0	1.8080	0	2.266	2.059	0	4.698	0	2.282
5	1.342	1.596	0.577	0.8407	0	1.756	0	2.089	2.326	0	4.918	0	2.115
6	1.225	1.410	0.483	0.8686	0.026	1.711	0.030	1.970	2.534	0	5.078	0	2.004
7	1.134	1.277	0.419	0.8882	0.105	1.672	0.118	1.882	2.704	0.205	5.203	0.076	1.924
8	1.061	1.175	0.373	0.9027	0.167	1.638	0.185	1.815	2.847	0.387	5.307	0.136	1.864
9	1.000	1.094	0.337	0.9139	0.219	1.609	0.239	1.761	2.970	0.546	5.394	0.184	1.816
10	0.949	1.028	0.308	0.9227	0.262	1.584	0.284	1.716	3.078	0.687	5.469	0.223	1.777
11	0.905	0.973	0.285	0.9300	0.299	1.561	0.321	1.679	3.173	0.812	5.534	0.256	1.744
12	0.866	0.925	0.266	0.9359	0.331	1.541	0.354	1.646	3.258	0.924	5.592	0.284	1.716
13	0.832	0.884	0.249	0.9410	0.359	1.523	0.382	1.618	3.336	1.026	5.646	0.308	1.692
14	0.802	0.548	0.235	0.9453	0.384	1.507	0.406	1.594	3.407	1.121	5.693	0.329	1.671
15	0.775	0.816	0.223	0.9499	0.406	1.492	0.428	1.572	3.472	1.207	5.737	0348	1.652
16	0.759	0.788	0.212	0.9523	0.427	1.478	0.448	1.552	3.532	1.285	5.779	0.364	1.636
17	0.0728	0.762	0.203	0.9951	0.445	1.465	0.466	1.534	3.588	1.359	5.817	0.379	1.621
18	0.707	0.738	0.194	0.9576	0.461	1.454	0.482	1.518	3.640	1.426	5.854	0.392	1.668
19	0.688	0.717	0.187	0.9599	0.477	1.443	0.497	1.503	3.689	1.490	5.888	0.404	1.596
20	0.671	0.697	0.180	9.9619	0.491	1.433	0.510	1.499	3.735	1.548	5.922	0.414	1.586
21	0.655	0.679	6.173	0.9638	0.504	1.424	0.523	1.477	3.778	1.606	5.950	0.425	1.575
22	0.640	0.662	0.167	0.9655	0.516	1.415	0.534	1.466	3.819	1.659	5.979	0.434	1.566
23	0.626	0.647	0.162	0.9670	0.527	1.407	0.545	1.455	3.858	1.710	6.006	0.443	1.557
24	0.612	0.632	0.157	0.9684	0.538	1.399	0.555	1.445	3.895	1.759	6.031	0.452	1.548
25	0.600	0.610	0.153	0.9696	0.548	1.392	0.565	1.435	3.931	1.804	6.058	0.459	1.541

1·34. Construct *c*-chart for the following data :

Airplane number	*No. of missing rivets*	*Airplane number*	*No. of missing rivets*	*Airplane number*	*No. of missing rivets*
201	8	210	12	218	14
202	16	211	23	219	11
203	14	212	16	220	9
204	19	213	9	221	10
205	11	214	25	222	22
206	15	215	15	223	7
207	8	216	9	224	28
208	11	217	9	225	9
209	21				

Is the process in control ? If not, recommend the control limits for future use.

1·35. In the final inspection of cars manufactured in a factory each car is checked for minor defects. These defects do not influence the acceptance or rejection of the cars. Rather they require a certain amount of additional labour to satisfy the consumer. A brief study was made to see if the number of minor defects was relatively constant and under control. The results of first twelve units checked are given below.

Unit No. (i) :	1	2	3	4	5	6	7	8	9	10	11	12
No. of defects (c_i) :	4	3	7	4	5	5	4	5	7	8	6	7

Discus the control chart for this process stating the underlying assumptions and give your conclusions.

If the manufacturer wants to estimate the proportion of manufactured cars that are free from defects, how can he use the above data to obtain a minimum variance unbiased estimate ? How does it compare with the maximum likelihood estimate ?

1·36. An inspector with 80 per cent efficiency (*i.e.*, for him the probability of classifying a defective as a non-defective is 0·20) uses the sampling plan $n = 100, c = 1$. Plot the effective *OC* curve.

1·37. Construct a single sampling plan for attributes, given the following data :

A.Q.L. = 0·05, Producer's risk = 0·05, L.P.T.D. = 0·20, Consumer's risk = 0·10

1·38. Explain how you will draw the O.C. curve for a single sampling plan with sample size 5 if the acceptance number $c = 2$, assuming the lot size to be large.

1·39. For the single sampling plan : $N = 2000, n = 100, c = 2$

(*i*) Find P_a (Probability of accepting the lot) when lot quality $p = 0·005, 0·01, 0·05, 0·10$.

1·40. A double sampling plan is given by $N = 2{,}000$

$$n_1 = 100 \quad ; \quad n_2 = 150 \quad ; \quad c_1 = 1 \quad ; \quad c_2 = 4$$

The lots rejected by the plan are 100% inspected and all the defectives found are replaced by good ones.

(*i*) Draw OC curve for this plan, calculating at least 8 points and state the approximations used, if any.

(*ii*) Draw the ASN curve for the same plan.

(*iii*) Find the average amount of inspection per lot for lots having 3% defetives.

1·41. Interpret the given double sampling pan, given $n_1 = 35, c_1 = 0, n_2 = 55, c_2 = 3$ and $N = 1{,}000$, with usual notations.

1·42. Between a single sampling with $n = 20, c = 2$ and a double sampling with $n_1 = 10, c_1 = 0, n_2 = 10, c_2 = 2$, cannot be said that the second inspection scheme is always more economical than the first ? Give reasons justifying your answer.

1·43. (*a*). Draw OC curve, AOQ curve and obtain AOQL. Also draw ASN curve for the following single sampling plan :

$$N = 2000, n = 150 \text{ and } c = 3$$

(*b*) A product in lots is submitted for inspection and the following double sampling plan is used :

$$n_1 = 50, \quad c_1 = 0, \quad n_2 = 60, \quad c_2 = 3$$

Assuming that the lot is very large compared to sample sizes, derive $L(p)$, the probability for acceptance.

(b) Compute separate control limits for any points that seem to be required by you.

(c) Based on this month's record, what would you recommend as the value of p' to use for the following month ?

Date	Number Inspected	Fraction Defective	Date	Number Inspected	Fraction Defective	Date	Number Inspected	Fraction Defective
Nov. 2	531	0·0471	12	2150	0·0270	23	1700	0·0288
3	1393	0·0445	13	2417	0·0476	24	2214	0·0307
4	1422	0·0428	14	2549	0·0451	25	2394	0·0343
5	1500	0·0487	16	2331	0·0322	26	1197	0·0468
6	1250	0·0368	17	2009	0·0403	27	850	0·0318
7	2000	0·0290	18	2198	0·0392	28	848	0·0353
9	685	0·0408	19	2271	0·0295	30	850	0·0388
10	2385	0·0373	20	1948	0·0210			
11	2150	0·0414	21	2150	0·0358			

1·27. Construct appropriate control chart :

Lot Number	:	1	2	3	4	5	6	7	8	9	10
Number inspected	:	500	400	300	150	600	450	750	800	900	1000
Number of defectives	:	25	42	35	16	15	40	72	81	82	100

Estimate the process average fraction defective.

1·28. A *c*-chart is to be introduced. From a previous study the average number of defects per item is found to be 4·84. Find the 3σ and 2σ control limits for the *c*-chart.

1·29. It is desired to maintain *c*-chart to control the average number of defects, c' at 4. Obtain the usual control limits for *c*-chart. Give the appropriate usual instructions to its users along with the specimen of the *c*-chart required for the above purpose. Find probability that your control limits detect that c' has, in fact, changed from 4 to 8.

1·30. The number of defects in 20 pieces of cloth each of 100 metres length is given below :

1, 3, 3, 1, 6, 4, 3, 7, 10, 2, 2, 6, 4, 3, 2, 1, 5, 6, 4.

Draw the appropriate chart and say whether the process can be considered to be in control.

1·31. The following number of defects were found on articles being produced, when inspected eight times in a day on three days :

2, 4, 7, 3, 1, 4, 8, 9 ; 5, 3, 7, 11, 6, 4, 9, 9 ; 6, 4, 3, 9, 7, 4, 7, 12

Draw the control chart and comment on your findings. If the times of inspection on the first day were 0800, 0905, 1010, 1100, 1230, 1335, 1420 and 1530 hours, comment on their irregularity of inspection spacing.

1·32. Draw a suitable control chart for the following data pertaining of the number of coloured threads (considered as defects) in 15 pieces of cloth in a certain make of synthetic fibre and state your conclusions :

7, 12, 3, 20, 21, 5, 4, 3, 10, 8, 0, 9, 6, 7, 20.

1·33. 20 samples of cloth, each of equal length and width were examined in order to launch a quality control programme. The number non-conformities observed per sample are shown below :

Sample No.	:	1	2	3	4	5	6	7	8	9	10
No. of Errors	:	1	4	4	1	6	3	5	10	7	3
Sample No.	:	11	12	13	14	15	16	17	18	19	20
No. of Errors	:	2	5	9	8	4	2	7	2	6	4

Draw the control chart. Is the process in control ?

[**Ans.** $UCL = 11·12$, $LCL = 0$]

What can be concluded about the behaviour of the process during the past month ? What limits are appropriate for controlling next month's production ?

(*b*) Given the following data on number of defectives in samples of sizes 100, construct the appropriate control chart. Interpret the results.

Sample No.	*Number Defective (d)*	*Sample No.*	*Number Defective (d)*	*Sample No.*	*Number Defective (d)*
1	3	15	5	29	6
2	1	16	8	30	5
3	4	17	2	31	5
4	4	18	3	32	6
5	4	19	5	33	4
6	6	20	4	34	9
7	5	21	3	35	6
8	5	22	4	36	4
9	2	23	6	37	3
10	4	24	4	38	1
11	3	25	3	39	2
12	4	26	5	40	1
13	4	27	4		
14	3	28	7		

1·24. The Ramington Corporation produces synthetic and natural gut castings for a process meat packer. Natural gut materials are visually inspected upon receipt, graded, and tested under pressure on a special device to ensure a specified strength before shipping to the meat packer. During the past 25 lots of 500 castings, each have been subjected to 100% inspection. A total of 1,000 castings burst during test :

(*i*) Find 3σ limits for a control chart for *p*.

(*ii*) Assuming that all points fall within these limits, what is your estimate of the process average fraction defective p' ?

[Ans. (*i*) $\bar{p} = \dfrac{1{,}000}{25 \times 500} = 0{\cdot}08$; $3\sigma_p = 3\sqrt{\dfrac{0{\cdot}08\,(1-0{\cdot}08)}{500}} = 0{\cdot}036$; $UCL_p = 0{\cdot}116$; $LCL_p = 0{\cdot}44$,

(*ii*) Since all the points fall within these limits, the process is under control and $\hat{p}' = \bar{p} = 0{\cdot}08$.]

1·25. In the manufacture of certain special duty transformers, units are required to meet a number of specifications related to temperature rise, output voltages, voltage and current ripple, on-off recovery times, etc. Approximately 200 units are produced and subjected to a final inspection daily. At the end of 20 working days, 230 units have been rejected out of 4,150 units produced and inspected.

(*i*) Determine 3σ trial control limits for a *p*-chart based on the estimated average daily production of 200 units.

(*ii*) Only one point on the control chart falls outside-limits. On that day, 30 nonconforming units were found in 200 units inspected. Investigation uncovered the act that a voltage pot setting was being incorrectly adjusted. What aimed at values of p_0' and control limits would you recommend for the following period based on average daily production of 200 units ?

[Ans. (*i*) $\bar{p} = \dfrac{230}{4{,}150} = 0{\cdot}055$, $3\sigma_p = 0{\cdot}048$; $UCL_p = 0{\cdot}055 + 0{\cdot}048 = 0{\cdot}103$; $LCL_p = 0{\cdot}055 - 0{\cdot}048 = 0{\cdot}007$

(*ii*) $\bar{p}' = \dfrac{230-30}{4{,}150-200} = 0{\cdot}051$; $3\sigma_p' = 0{\cdot}047$; $UCL_p = 0{\cdot}051 + 0{\cdot}047 = 0{\cdot}098$; $LCL = 0{\cdot}051 - 0{\cdot}047 = 0{\cdot}004$.]

1·26. The following table gives the results of daily inspection of a vacuum tube. The standard value of fraction defective p' established at the start of the month was 0·04. The estimated daily average production was 1600 tubes.

(*a*) Establish a single set of control limits based on these figures and plot a control chart.

shift of μ to 425 within two samples after a shift occurs are 99 per cent. For the same chart, what are the chances of detecting a shift of μ from 400 to 380 within two samples.

1·17. An item is made in lots of 200 each. The lots are given 100% inspection. The record sheet for the first 25 lots inspected showed that a total of 75 items were defective.

(*i*) Determine the trial control limits for an *np*-chart showing the number of defectives in each lot.

(*ii*) Assume that all points fall within the control limits. What is your estimate of the process average fraction defective p' ?

(*iii*) If this p' remains unchanged, what is the probability that the 26th lot will contain exactly 7 defectives ?, that it will contain 7 or more defectives ?

1·18. A *p*-chart indicates that the average is 0·2. If 50 items are inspected each day, what is the probability of detecting a shift of 0·04.

(*i*) on the first day after the shift, and (*ii*) by the end of the third day after the shift.

1·19. A process produces 3% defectives. If $n = 4$, what would be 3σ limits for the *p*-chart ? In this chart, what is the probability of detecting a shift to 5% defectives in the next sample of 400 units ?

1·20. Discuss the theoretical basis of *p* and *np*-charts. Given that the process fraction defective p' is 0·2 and *n* is 25, calculate the control limits for *p* and *np*-charts.

1·21. Each day a sample of 50 items from a production process was examined. The number of defectives found in each sample was as follows :

6, 2, 5, 1, 2, 2, 3, 5, 3, 4, 12, 4, 4, 1, 3, 5, 4, 1, 4, 3, 5, 4, 2, 3.

Draw a suitable control chart and check for control. What control limits would you suggest for subsequent use ?

1·22. In a factory producing spark plugs, the number of defectives found in the inspection of 20 lots of 100 each is given below :

INSPECTION DATA ON COMPLETED SPARK PLUGS
(2000 spark plugs in 20 lots of 100 each)

Lot No.	*No. of Defectives*	*Fraction Defective*	*Lot No.*	*No. of Defectives*	*Fraction Defective*	*Lot No.*	*No. of Defectives*	*Fraction Defective*
1	5	0·050	8	3	0·030	15	3	0·030
2	10	0·100	9	3	0·030	16	4	0·040
3	12	0·120	10	5	0·050	17	5	0·050
4	8	0·080	11	4	0·040	18	8	0·080
5	6	0·060	12	7	0·070	19	6	0·060
6	4	0·040	13	8	0·080	20	10	0·100
7	6	0·060	14	2	0·020	Total	120	

Find out the lower control limit and the upper control limit from data. Is the above data within control limits ?

1·23. (*a*) Given the following data :

Sample No.	*Sample Size*	*No. of Defectives*	*Sample No.*	*Sample Size*	*No. of Defectives*	*Sample No.*	*Sample Size*	*No. of Defectives*
1	200	3	9	200	3	16	200	9
2	200	1	10	200	2	17	200	3
3	200	0	11	200	1	18	200	1
4	200	2	12	200	3	19	200	0
5	200	4	13	200	6	20	200	2
6	200	1	14	200	8	21	200	3
7	200	2	15	200	5	22	200	1
8	200	0						

Construct the appropriate control chart for the process represented by the above data collected in the past months.

$\Sigma\sigma = 109{\cdot}5$. Compute the values of the 3-sigma limits for the $\overline{X}$ and σ-Charts, and estimate the values of σ′ on the assumption that the process is in statistical control.

[**Hint.** $\overline{\overline{X}} = \frac{\Sigma\overline{X}}{12} = 108{\cdot}917, \quad \overline{\sigma} = \frac{\Sigma\sigma}{12} = 15{\cdot}958$; $UCL_{\overline{X}} = \overline{\overline{X}} + A_1\,\overline{\sigma} = 122{\cdot}0,\ LCL_{\overline{X}} = 95{\cdot}83$;

$UCL_\sigma = B_4\,\overline{\sigma} = 25{\cdot}05$; $LCL_\sigma = B_3\,\overline{\sigma} = 6{\cdot}86$ For $n = 15$, $c_2 = 0{\cdot}9490$ ∴ $\sigma' = \frac{\overline{\sigma}}{c_2} = \frac{15{\cdot}958}{0{\cdot}9490} = 16{\cdot}81$]

1·12. In order to meet government regulations, the contained weight of a product must at least equal the labeled weight 98% of the time. Control charts for $\overline{X}$ and σ are maintained on the weight in ounces of the contents using a sub-group size of 10. After 20 sub-groups, $\Sigma\overline{X} = 731{\cdot}4$ and $\Sigma\sigma = 9{\cdot}16$. Compute 3σ control limits for $\overline{X}$ and σ and estimate the value of σ′ assuming the process is in statistical control. If the label weight is 36 oz, and assuming the process generates a normal distribution, does it meet governmental requirement ?

[**Hint.** $\overline{\overline{X}} = 36{\cdot}57$; $\overline{\sigma} = 0{\cdot}458$; $UCL_{\overline{X}} = 37{\cdot}04$, $LCL_{\overline{X}} = 36{\cdot}10$; $UCL_\sigma = 0{\cdot}788$; $LCL_\sigma = 0{\cdot}128$; $\sigma' = \frac{\overline{\sigma}}{c_2} = 0{\cdot}495$

Product below the specifications $= 100\,P(X \le 36) = 100\,P(Z \le -1{\cdot}149) = 100\,(0{\cdot}5 - 0{\cdot}3748) = 12{\cdot}5\%$

Hence it does not meet the government requirement.]

1·13. The diameter of one end of a gyro drive shaft is required within 1,140 ± 10. The control charts for $\overline{X}$ and R are initiated. After 30 sub-groups of 5 shafts each have been examined, $\Sigma\overline{X} = 34{,}290$ and $\Sigma R = 330$:

(*i*) Estimate the mean μ′ and standard deviation σ′ of the process assuming that it is in statistical control.

(*ii*) Determine the 3-sigma limits for $\overline{X}$ and R-charts.

(*iii*) Determine the natural specification limits of the process.

[**Hint.** (*i*) $\overline{\overline{X}} = (34{,}290/30) = 1,143$, $\overline{R} = 330/30 = 11$; $\hat{\mu}' = \overline{\overline{X}} = 1143$ and $\sigma' = (\overline{R}/d_2 = 11/2{\cdot}326 = 4{\cdot}73$

(*ii*) $UCL_{\overline{X}} = 1149{\cdot}38$; $LCL_{\overline{X}} = 1136{\cdot}62$; $UCL_R = 23{\cdot}21$, $LCL_R = 0$.

(*iii*) Upper natural specification limit $= \overline{\overline{X}} + 3\sigma' = 1157{\cdot}19$;

Lower natural specification limit $= \overline{\overline{X}} - 3\sigma' = 1128{\cdot}81$.]

1·14. The specification limits for a certain dimension are 3·5100± 0·0050 inches. Control charts for $\overline{X}$ and R indicate that the $\overline{X}$-chart shows lack of control but the R-chart always shows control. From the R chart, estimate of σ′ is 0·0010. If the aimed at process average $\overline{X}'$ is to be 3·51, what should be the upper control limit for $\overline{X}$ with a sub-group of size 4 ? What should be the upper reject limit on the $\overline{X}$ chart ?

1·15. The measurement X on a manufactured unit is assumed to follow a normal distribution with mean μ and variance one. The process is under control if $\mu = 0$ and is not under control if $\mu \ne 0$. Construct a control chart for the sample mean $\overline{X}$ based on random sub-samples of size 4 and using 3σ limits. Construct another control chart such that for every sub-sample, it provides you with a uniformly most powerful unbiased level α, $0 < \alpha < 1$, test of the hypothesis $\mu = 0$ against the alternative $\mu \ne 0$. Which control chart would you prefer and why ?

1·16. Derive a control chart to test $\mu = 400$, given that $\sigma = 30$, such that the chances of a point going of the the control limits when the process really is in control are 1 in 20 while the chances of detecting a for 25 samples of 4 each. A milling machine gave an average of 4·875 inches and average range of 0·001 inch for 20 samples of 4 each.

[**Hint.** Specification Limits : $U = 4{\cdot}875 + 0{\cdot}001 = 4{\cdot}876$; $L = 4{\cdot}875 - 0{\cdot}001 = 4{\cdot}874$

For broach : $\overline{\overline{X}} = 4{\cdot}877$; $\overline{R} = 0{\cdot}0005$, $\sigma' = \dfrac{{\cdot}0005}{2{\cdot}059} = 0{\cdot}00024$, $UCL = 4{\cdot}87784$, $LCL = 4{\cdot}87628$.

For Milling Machine : $\overline{\overline{X}} = 4{\cdot}875$; $\overline{R} = 0{\cdot}001$; $\sigma' = \dfrac{0{\cdot}001}{2{\cdot}059} = 0{\cdot}00048$; $UCL = 4{\cdot}87644$; $LCL = 4{\cdot}87356$.

From above results, it is evident that in the case of broach, most of the product will be outside the specification limits while in the case of milling machine most of the product will be within the specification limits. Hence milling operation is perferable.]

1·10. The following data (pertaining to two sub-groups of size 4) is from two different machines which are supposed to be alike. Plot the necessary charts to show whether their product would support this assumption. If they do not support this assumption, does this prove the machines are not essentially alike ?

Sub-group No.	*Machine 1* *Average*	*Range*	*Sub-group No.*	*Machine 2* *Average*	*Range*
1	2·77	0·06	1	2·53	0·12
2	2·70	0·29	2	2·67	0·10
3	2·78	0·19	3	2·66	0·17
4	2·67	0·12	4	2·57	0·25
5	2·75	0·34	5	2·60	0·05
6	2·77	0·23	6	2·60	0·24
7	2·75	0·17	7	2·70	0·30
8	2·73	0·06	8	2·56	0·04
9	2·76	0·23	9	2·70	0·19
10	2·63	0·20	10	2·67	0·08
11	2·73	0·17	11	2·60	0·11
12	2·73	0·28	12	2·63	0·14
13	2·74	0·26	13	2·71	0·24
14	2·72	0·13	14	2·63	0·31
15	2·73	0·13	15	2·75	0·17

[**Hint.** *For Machine 1* :

$\overline{R} = 0{\cdot}191$; $\overline{\overline{X}} = 2{\cdot}73$; $\sigma' = 0{\cdot}0927$; $UCL_{\overline{X}} = 2{\cdot}87$; $LCL_{\overline{X}} = 2{\cdot}59$

For Machine 2 :

$\overline{R} = 0{\cdot}181$; $\overline{\overline{X}} = 2{\cdot}64$; $\sigma' = 0{\cdot}088$; $UCL_{\overline{X}} = 2{\cdot}77$; $LCL_{\overline{X}} = 2{\cdot}51$

Plot the $\overline{X}$-charts for both the machines. From the $\overline{X}$-Charts, it will be evident that these two machines are not identical; Machine 2 shows somewhat rising trend. Assuming that variation in the dimensions is due to machines only, σ′ of the universes produced by these machines are 0·0927 and 0·088 respectively which are different and therefore the machines are not essentially alike.]

1·11. Control charts for $\overline{X}$ and σ are maintained on the reaching strength in pounds in a certain destructive test of a particular type of ceramic insulator used in vacuum tubes. The sub-group size is 15. The values of $\overline{X}$ and σ are computed for each sub-group. After 12 sub-groups, $\Sigma\,\overline{X} = 1{,}307$ and

1·5. The following data shows the values of samples mean $\overline{X}$ and the range R for ten sample of size 5 each. Calculate the values for central line and control limits for mean chart and range chart, and determine whether the process is in control.

Sample No.	:	1	2	3	4	5	6	7	8	9	10
Mean ($\overline{X}$)	:	11·2	11·8	10·8	11·6	11·0	9·6	10·4	9·6	10·6	10·0
Range (R)	:	7	4	8	5	7	4	8	4	7	9

(Conversion factors for $n = 5$ are $A_2 = 0{\cdot}577$, $D_3 = 0$, $D_4 = 2{\cdot}115$.)

1·6. A fair percentage of a certain product requires costly rework operations to change a certain quality characteristic. Rework is possible whenever the quality characteristic falls above the upper specification limit. If the value falls below the lower specification limit, the product must be scrapped. $\overline{X}$ and R charts have been maintained for 50 sub-groups of 5 each. The specifications for the quality characteristic are 119 ± 10. The process appears to be in statistical control with $\overline{X}'$ as 124 and σ' as 5. On the assumption that the quality characteristic is normally distributed, approximately what percentage of defective articles is being produced ? How much of this can be reworked ?

1·7. The following table gives the values of mean and range of a sample of five observations :

Sample No.	*Mean*	*Range*	*Sample No.*	*Mean*	*Range*	*Sample No.*	*Mean*	*Range*
1	4·17	·14	8	4·25	·11	15	4·50	·22
2	4·15	·30	9	4·54	·58	16	4·07	·09
3	4·08	·20	10	4·54	·22	17	4·33	·17
4	4·26	·26	11	4·35	·62	18	4·61	·20
5	4·13	·10	12	4·54	·23	19	4·32	·20
6	4·22	·24	13	4·31	·28	20	4·57	·40
7	4·33	·65	14	4·28	·38			

(*i*) Find an estimate of σ' from the above data.

(*ii*) What are the control limits for $\overline{X}$-chart and R-chart ? What can you conclude about the process ?

(*iii*) If the specifications are $4{\cdot}35 \pm 0{\cdot}5$, what can you say about the process ?

1·8. The samples of 5 are taken from a manufacturing process and certain quality characteristic is measured. The $\overline{X}$ and R values are computed for each sub-group. After 20 subgroups, $\Sigma \overline{X} = 0{\cdot}87632$ cms and $\Sigma R = 0{\cdot}2410$ cms.

(*a*) Compute the control limits for the control charts.

(*b*) The above samples were taken every 15 minutes in order of production. The production rate was 400 per hour and the specification limits were 0·0430 and 0·0460 cm.

(*i*) What is the per cent defective of the above process operating at the levels indicated ?

(*ii*) What would happen if the process average should shift to 0·04315 ?

(*iii*) What is the probability that you would catch such a shift on your control chart on the first sample following the shift ?

(*iv*) How many samples would you have to take to have a chance of approximately 0·95 of catching this shift on at least one of these samples ?

(*v*) Find the number of defective articles produced by the process in these samples.

1·9. The head of an automobile engine must be machined so that both the surface that meets the engine block and the surface that meets the valve covers are flat. The surfaces must also be 4·875 inches ± 0·001 inch apart. Presuming that the valve cover side of the head is finished correctly, compare the capabilities of two processes for performing the finishing of the engine block side of the head. A broach set to do the job gave on average thickness of 4·877 inches with an average range of 0·0005 inch

13	0.838	0.822	0.835	0.830	0.830	0.8310	0.016
14	0.815	0.832	0.831	0.831	0.838	0.8294	0.023
15	0.831	0.833	0.831	0.834	0.832	0.8322	0.003
16	0.830	0.819	0.819	0.844	0.832	0.8288	0.025
17	0.826	0.839	0.842	0.835	0.830	0.8344	0.016
18	0.813	0.833	0.819	0.834	0.836	0.8270	0.023
19	0.832	0.831	0.825	0.831	0.850	0.8338	0.025
20	0.831	0.838	0.833	0.831	0.833	0.8332	0.007
21	0.823	0.830	0.832	0.835	0.835	0.8310	0.012
22	0.835	0.829	0.834	0.826	0.828	0.8304	0.009
23	0.833	0.836	0.831	0.832	0.832	0.8328	0.005
24	0.826	0.835	0.842	0.832	0.831	0.8332	0.016
25	0.833	0.823	0.816	0.831	0.838	0.8282	0.022
26	0.829	0.830	0.830	0.833	0.831	0.8306	0.004
27	0.850	0.834	0.827	0.831	0.835	0.8354	0.023
28	0.835	0.846	0.829	0.833	0.822	0.8330	0.024
29	0.831	0.832	0.834	0.826	0.833	0.8312	0.008

(*b*) Using the data below, calculate limits and plot the $\overline{X}$ and R-charts. Apply the standard tests for unnatural patterns and discuss the results. (The sample size is n = 5.)

Sample No.	$\overline{X}$	R	*Sample No.*	$\overline{X}$	R	*Sample No.*	$\overline{X}$	R
1	1.444	0.09	18	1.424	0.05	35	1.457	0.09
2	1.427	0.08	19	1.434	0.05	36	1.444	0.09
3	1.464	0.08	20	1.414	0.09	37	1.432	0.05
4	1.455	0.08	21	1.406	0.07	38	1.438	0.05
5	1.462	0.10	22	1.418	0.14	39	1.404	0.10
6	1.448	0.05	23	1.438	0.09	40	1.409	0.05
7	1.454	0.04	24	1.416	0.07	41	1.400	0.07
8	1.446	0.08	25	1.419	0.06	42	1.425	0.09
9	1.437	0.12	26	1.406	0.08	43	1.405	0.07
10	1.471	0.11	27	1.428	0.06	44	1.419	0.10
11	1.438	0.09	28	1.430	0.06	45	1.410	0.07
12	1.438	0.05	29	1.421	0.07	46	1.420	0.05
13	1.415	0.12	30	1.434	0.07	47	1.414	0.05
14	1.428	0.12	31	1.408	0.05	48	1.426	0.11
15	1.425	0.08	32	1.414	0.08	49	1.386	0.06
16	1.440	0.09	33	1.410	0.03	50	1.387	0.08
17	1.430	0.05	34	1.406	0.06			

1·4. In order to determine whether or not a production of bronze casting is in control, 20 sub-groups of size 6 are taken. The quality characteristic of interest is the weight of the castings and it is found that $\overline{\overline{X}}$ is 3·126 gm and $\overline{R}$ = 0·009 gm.

(*i*) Estimate the standard deviation of the weight of castings.

(*ii*) Assuming that the process is in control, find upper and lower control limits for the sub-groups means.

(*iii*) Assuming that the process is in control, find upper and lower control limits for the sub-group ranges.

[For $n = 6$, $d_2 = 2{\cdot}534$]

ASSORTED REVIEW PROBLEMS

1·1. (*a*) Control charts for $\overline{X}$ and R are maintained on the tensile strength in pounds of a certain yarn. The sub-group size is 5. The values of $\overline{X}$ and R are computed for each sub-group. After 25 sub-groups, $\Sigma\overline{X} = 514{\cdot}8$ and $\Sigma R = 120{\cdot}0$. Compute the values of 3-sigma limits for the $\overline{X}$ and R-charts and estimate value of σ on the assumption that process is in statistical control.

(*b*) A machine is manufacturing mica discs with specified thickness between 0·008″ and 0·015″. Samples of size 4 are drawn every hour and their thickness in units (1 unit = 0·001″) are recorded in the adjoining Table.

Sample No.	*Thickness of mica discs. (1 unit = ·001″)*			
1	14	8	12	12
2	11	10	13	8
3	11	12	16	13
4	15	12	14	11
5	10	10	8	8

For the above data, set up an R-chart and an $\overline{X}$-chart. Plot the observed points and comment on the same.

1·2. Construct a control chart for mean and range for the following data on the basis of fuses, samples of 4 being taken every hour.

Sample No.	*Observations*				*Sample No.*	*Observations*			
1	27	23	36	24	14	28	30	17	23
2	30	17	27	32	15	44	32	22	41
3	21	44	22	28	16	26	42	35	28
4	40	21	29	24	17	38	40	51	32
5	51	34	17	10	18	26	28	34	39
6	33	30	28	22	19	42	38	52	36
7	30	22	18	12	20	30	32	39	45
8	35	48	20	47	21	23	44	48	33
9	20	34	15	42	22	28	34	39	44
10	22	50	45	41	23	25	29	40	33
11	34	22	36	44	24	30	38	44	32
12	32	48	32	33	25	38	27	39	22
13	34	32	28	38					

1·3. (*a*) Construct and interpret the appropriate control charts for the following data :

Sample No.	*Individual Value*					$\overline{X}$	R
	X_1	X_2	X_3	X_4	X_5		
1	0.831	0.829	0.836	0.840	0.826	0.8324	0.014
2	0.834	0.826	0.831	0.831	0.831	0.8306	0.008
3	0.836	0.826	0.831	0.822	0.816	0.8262	0.020
4	0.833	0.831	0.835	0.831	0.833	0.8326	0.004
5	0.830	0.831	0.831	0.833	0.820	0.8290	0.013
6	0.829	0.828	0.828	0.832	0.841	0.8316	0.013
7	0.835	0.833	0.829	0.830	0.841	0.8336	0.012
8	0.818	0.838	0.835	0.834	0.830	0.8310	0.020
9	0.841	0.831	0.831	0.833	0.832	0.8336	0.010
10	0.832	0.828	0.836	0.832	0.825	0.8306	0.011
11	0.831	0.838	0.844	0.827	0.826	0.8332	0.018
12	0.831	0.826	0.828	0.832	0.827	0.8288	0.006

preparation of a single sampling inspection plan under any one of the two types of protection on the basis of attribute sampling.

25. (*a*) What do you understand by an acceptance sampling plan ?

(*b*) To devise a single sampling plan by attributes from the basic information provided, outline the steps to be followed in arriving at the desired sampling plan.

(*c*) How is the efficiency of sampling plan judged ? Illustrate with an example.

26. What are single sampling plan and double sampling plan ? Discuss the relative merits and demerits of single and double sampling plans.

27. In a single sampling plan of attributes with lot size N, sample size n and allowable defectives c, how will you obtain the probability of acceptance of the lot if the lot fraction defective is p ?

How will you modify the above expression using : (*i*) Binomial approximation and (*ii*) Poison approximation ?

28. A sample of size n is drawn from the items produced by a certain machine which is known to produce on an average $100 \times p\%$ defectives. In order to ensure the quality of the item to be purchased by a consumer, it is decided that a full day's production will be purchased by him if the sample selected does not contain more than C defective items. Otherwise, all the items produced on the day are to be inspected and all defectives detected are to be replaced by good ones and then the good items only be supplied to the consumer. Derive expressions for : (*i*) *OC*, (*ii*) *AOQ*, (*iii*) *ASN*, and (*iv*) *ATI*.

29. Describe the single sampling plan for acceptance sampling, deriving expressions for the producer's and consumer's risks, and show that approximately

$$\text{ATI} = n + (N - n)\left[1 - \sum_{x=0}^{c}\left\{e^{-n\bar{p}}\left(n\bar{p}\right)^x \big/ x\,!\right\}\right]$$

the notations being usual. Discuss how the parameters are determined.

30 (*a*). What is Average Sample Number (ASN) and Average Total Inspection (ATI). Explain the method of their calculation for single sampling plan. Why are ASN and ATI calculated ?

(*b*) Explain clearly how ASN and ATI curves are obtained for a sampling plan by attributes when acceptance rectification scheme is in use.

31. (*a*) What is sampling inspection ? Distinguish between the rectifying and the non-rectifying types.

(*b*) (*i*) Explain clearly how one is led to AOQL, explaining the various intermediary concepts of acceptance sampling.

(*ii*) Find AOQL if acceptance/rectification scheme of inspection is adopted.

32. (*a*) Assuming that the number of defective items follows Poisson law, how would you determine the most economical single sampling plan given N (lot size), Average Outgoing Quality Limit (AOQL) and the process average quality ($\bar{p}$).

(*b*) Given lot size (N), LTFD (p_t), Consumer's risk (P_c) and process average quality ($\bar{p}$) derive the most economical single sampling plan for attributes for acceptance purposes.

33. Explain the concepts of producer's and consumer's risk in sampling inspection schemes. Define the average sample number and the average outgoing quality in the case of Double Sampling Inspection and indicate their usefulness in choosing a sampling scheme.

34. (*a*) Explain and bring out the distinction between Acceptance Quality Level (A.Q.L.) and Average Outgoing Quality Limit (A.O.Q.L.).

Describe the method of double sampling plan and derive its O.C., A.O.Q., A.S.N. and A.T.I. (Average Total Inspection).

(*b*) Explain single and double sampling inspection plans in quality control and discuss their relative merits and demerits.

35. Large lots are being submitted for inspection by attributes to a consumer who considers a lot as acceptable if its proportion defective does not exceed p_0 and unacceptable if the same exceeds or equals p_t ($p_1 < p_0$). Starting with the general results of sequential sampling, derive a suitable inspection plan to protect both the producer's and consumer's risks. Give a graphical description for working out the plan. Also indicate how you will draw the OC curve of the plan.

(c) Discuss the construction of p-chart when all samples are of same size. How is the procedure modified for variable sample size ? Discuss two methods in this case.

13. Explain the basis and working of a control chart to control the fraction defective when the number of articles inspected varies. State the important steps involved while establishing the control limits for future production giving the practical way of overcoming the difficulties that arise due to varying inspection number.

14. Which of the control charts are used for sampling by attributes ? Give practical examples from actual life situations where these charts can be used. Indicate the distribution on which their control limits are based and mention these limits.

15. Explain how you would proceed to draw control chart/charts when the data available are for a qualitative characteristic and the sizes of the samples drawn are different.

16. Under what conditions would you consider it economical to use the p-chart and the c-chart ? Examine the basis and the approximations involved in the use of the control charts.

17. Discuss the role of control charts with reference to specification, production and inspection. Explain some of the benefits of the charts for the fraction defective and the number of defects. Comment on the statement that even if the sample points are within control limits, the chart may indicate a tendency for lack of control.

18. Distinguish between defect and defective. Give some examples of defects for which the c-chart is applicable. How do you calculate control limits for a c-chart ? Discuss the assumptions and approximations involved in the calculations.

19. A company manufactures four models of radios and receives quite a few complaints from customers. In order to improve the quality of radios, the management decides to use control chart technique for the defects observed at the final testing. The number of radios manufactured daily is not constant.

Suggest a procedure, or set up control chart or charts for controlling the defects, giving clearly the assumptions made and the statistical concepts used.

20. (a) What is meant by specification limits and control limits. Does a process in statistical control ensure, that all the product will be within specifications ? Justify your statement by means of an example.

(b) Define process capability when the process shows a state of control. Discuss in detail the various courses of action often employed under the following situations :

(i) The process capability is greater than the specified tolerance.

(ii) The process capability is approximately equal to the specified tolerance.

(iii) The process capability is less than the specified tolerance.

(iv) When only a single specification limit is given, discuss its relationship with the process capability and the actions that can be taken under different settings of the machine.

21. (a) Explain the terms :

(i) Trial control limits, (ii) 3σ limits, (iii) Modified control limits and (iv) Specification limits.

(b) Explain the relation between the control limits, specification limits and reject limits. Also explain how the reject limits are constructed.

22. What are the types of control charts for inspection by attributes and how are they set up ? Let p_n be the probability of a mean of a sample of size n exceeding upper control limit when the population mean has shifted to $\mu' + k\sigma'$, find :

(i) The probability that at most m samples are to be taken for a point to go out of control, and

(ii) If we decide to pronounce lack of control when r points exceed upper control limit, the probability that m samples are to be taken for r points to exceed UCL.

23. (a) What do you understand by acceptance sampling procedure ? State its uses giving illustrations.

(b) Describe single sampling plan. Obtain OC and AOQ curve for this plan. Distinguish clearly between : (i) Producer's risk and Consumer's risk ; (ii) AQL and LTPD.

24. Explain the principles and the procedures of (i) Lot Quality Protection, and (ii) Average Quality Protection, assured to consumers by sampling inspection plans. Discuss the guidelines for the

DISCUSSION AND REVIEW QUESTIONS

1. (*a*) "Measured quality of manufactured product is always subject to a certain amount of variation as a result of chance. Some stable system of chance causes is inherent in any particular scheme of production and inspection. Variation within the system is inevitable. The Statistical Quality Control aims at discovery and rectification of reasons for variation outside this stable pattern." Elaborate.

(*b*) Give clearly meaning of each word of the term 'Statistical Quality Control'. Distinguish between 'process' and 'product' control. Does process control also ensure product control necessarily ?

2. (*a*) What do you understand by Statistical Quality Control (SQC) ? Discuss briefly its need and utility in industry. Discuss the causes of variation in quality.

(*b*) What is meant by process control in industrial statistics ?

(*c*) Examine the need for quality control techniques in production.

3. What is control chart ? Explain the basic principles underlying the control charts. Discuss the role of control charts in manufacturing processes.

4. Explain how a control chart helps to control the quality of a manufactured product. Describe the basis of a control chart. Distinguish clearly between the charts for variables and charts for attributes. Is it possible to use the latter for former ? If so, when and how ?

5. Comment on the statement "the aim of control chart technique is to stabilise the variability pattern of the product to keep a continual check so that the stability of the pattern is not disturbed for the worse and at the same time to seize upon any factor which may happen to improve upon the pattern". Illustrate your points with reference to (*i*) the *p*-chart, (*ii*) the *c*-chart, and (*iii*) the $\overline{X}$ and *R*-charts, and bring out the roles of the binomial, Poisson and normal distributions respectively in this connection.

6. Explain the justification for using the three sigma limits in the control charts irrespective of the actual probability distribution of the quality characteristic.

7. (*a*) Explain clearly the basis and working of control charts for mean and range. State the basis and assumptions on which $\overline{X}$ and *R*-charts are developed.

(*b*) Explain in detail the $\overline{X}$ and *R*-charts. What purposes do they serve ? What are their advantages over the *p* chart ?

Explain how to estimate σ from the mean range of samples of constant size drawn during a continuous production process. What are the other methods of estimating σ ?

8. (*a*) Explain the usefulness of *R*-chart. When is *s*-chart used in place of *R*-chart ?

(*b*) State the control limits of $\overline{X}$, *R* and *s*-charts. Also state how the standard deviation of the process can be estimated from $\overline{R}$ and $\overline{s}$.

9. Account for the following procedures in control chart analysis, stating clearly any assumptions involved :

(*i*) A sample of size 5 is ordinarily used in control chart,

(*ii*) The range is used instead of standard deviation, and

(*iii*) The three sigma limits are used for control limits instead of exact probability, whatever be the type of quality characteristics.

10. Explain the construction of a control chart for $\overline{X}$ when

(*i*) the standards for μ and σ are specified as μ' and σ' respectively, and

(*ii*) the standards are not specified.

11. Explain the main control charts for attributes and obtain their control limits. Discuss the advantages and disadvantages of control charts of variables and control charts of attributes.

12. (*a*) What do you understand by control chart for fraction defective ? Explain its construction. Give the theoretical distribution on which the control limits are based.

(*b*) How will you interpret a *p*-chart, particularly the points above the upper control limit and below the lower control limit.

Thus, the points of the ASN curve can be tabulated as follows:

p (*Submitted lot quality*)	*ASN* (*Average Sample Number*)	
0·00	20·0954	≃ 20·10
0·15	62·5262	≃ 62·53
0·2188	132·96	
0·30	61·28	
1·00	6·6172	≃ 6·62

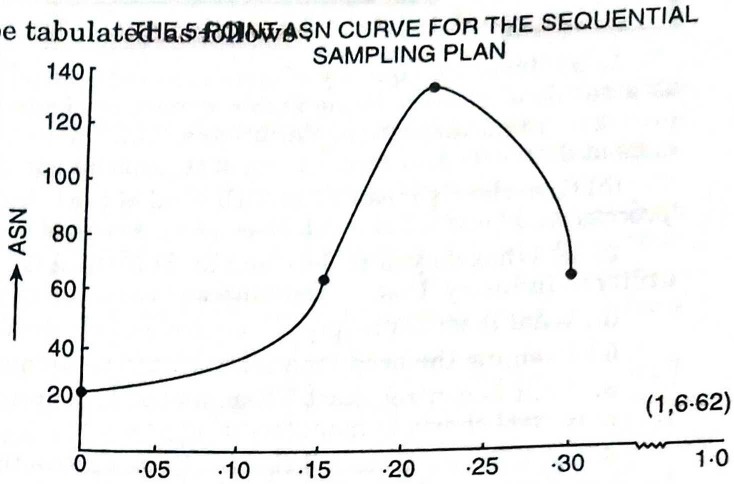

Fig. 1·27

Sample Size for arriving at a Decision

SEQUENTIAL SAMPLING PLAN

$a_m = -4{\cdot}3975 + 0{\cdot}2188m$; $r_m = 5{\cdot}1672 + 0{\cdot}2188m$

m	d_m	a_m	r_m	m	d_m	a_m	r_m
1	0	−4·1787	5·386	18	2	−0·4591	9·1056
2	0	−3·9599	5·6048	19	2	−0·2403	9·3244
3	1	−3·7411	5·8236	20	2	−0·0215	9·5432
4	1	−3·5223	6·0424	21	2	0·1973	9·7620
5	1	−3·3035	6·2612	22	2	0·4161	9·9808
6	1	−3·0847	6·4800	23	2	0·6349	10·1996
7	1	−2·8659	6·6988	24	2	0·8537	10·4184
8	1	−2·6471	6·9176	25	2	1·0725	10·6372
9	1	−2·4283	7·1364	26	2	1·2913	10·856
10	1	−2·2095	7·3552	27	3	1·5110	11·0748
11	1	−1·9907	7·5740	28	3	1·7289	11·2936
12	1	−1·7719	7·7928	29	3	1·9477	11·5124
13	1	−1·5531	8·0116	30	3	2·1665	11·7312
14	1	−1·3343	8·2304	31	3	2·3853	11·9500
15	1	−1·1155	8·4492	32	3	2·6041	12·1688
16	1	−0·8967	8·6680	33	3	2·8229	12·3876
17	2	−0·6779	8·8868	34	3	3·0417	12·6064

For $m = 34$, d_m lies outside a_m and r_m. In fact $d_m < a_m$ at $m = 34$. Hence, sequential sampling plan is terminated with the acceptance of the lot after inspecting the 34th item.

	L_1		L_2	
m	0	10	0	10
d_m	–4·3975	–2·2095	5·1672	7·3552

OC Curve. When $p = s = 0{\cdot}2188,\ P(p) = \dfrac{h_1}{h_1 + h_2} = \dfrac{4{\cdot}3975}{9{\cdot}5647} = 0{\cdot}46$

5 POINTS OF THE OC CURVE

p (*Submitted lot quality*)	$L(p)$ (*Probability of Acceptance*)
0	1·00
$p_0 = 0{\cdot}15$	$1 - \alpha = 0{\cdot}99$
$s = 0{\cdot}22$	0·46
$p_2 = 0{\cdot}30$	$\beta = 0{\cdot}02$
1·00	0

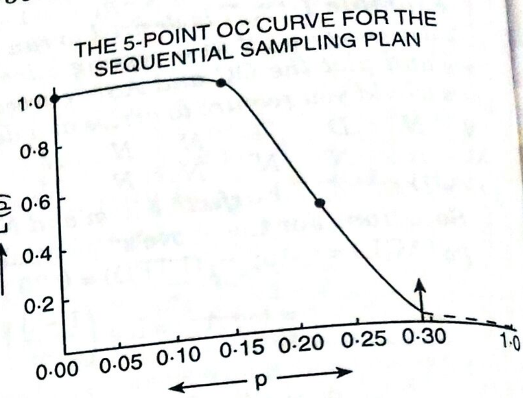

Fig. 1·26

The OC curve is drawn in adjoining *Fig. 1·26.*

ASN Curve. The general five points on the ASN curve are obtained as follows :

When $p = 0,\quad E(n) = \dfrac{b}{g_2} = \dfrac{1{\cdot}69461}{0{\cdot}08433} = 20{\cdot}095$

$p = p_0 = 0{\cdot}15,$

When $E(n) = \dfrac{(1-\alpha)\,h_1 - \alpha h_2}{s - p_0} = \dfrac{0{\cdot}99 \times 4{\cdot}3975 - 0{\cdot}01 \times 5{\cdot}1672}{0{\cdot}2188 - 0{\cdot}15} = 62{\cdot}5262$

When $p = s = 0{\cdot}2188,\ E(n) = \dfrac{h_1 h_2}{s(1-s)} = \dfrac{4{\cdot}3975 \times 5{\cdot}1672}{0{\cdot}2188 \times 0{\cdot}7812} = 132{\cdot}96$

For $p = p_1 = 0{\cdot}30,$

$$E(n) = \frac{(1-\beta)\,h_2 - \beta h_1}{p_1 - s}$$

$$= \frac{0{\cdot}98 \times 5{\cdot}1672 - 0{\cdot}02 \times 4{\cdot}3975}{0{\cdot}30 - 0{\cdot}2188} = 61{\cdot}28$$

For $p = 1,\ E(n) = \dfrac{a}{g_1} = \dfrac{1{\cdot}99123}{0{\cdot}30103} = 6{\cdot}61472$

From (2) and (3), we get

$$c_1 < p_1,\ \frac{1}{c_2} < \frac{1}{1-p_1} \qquad\Big|\qquad c_1 > p_0,\ \frac{1}{c_2} > \frac{1}{1-p_0}$$

$$\Rightarrow \frac{c_1}{c_2} < \frac{p_1}{1-p_1} \qquad\Big|\qquad \Rightarrow \frac{c_1}{c_2} > \frac{p_0}{1-p_0}$$

Hence, $\dfrac{p_0}{1-p_0} < \dfrac{c_1}{c_2} < \dfrac{p_1}{1-p_1} \Rightarrow \dfrac{p_0}{1-p_0} < \dfrac{s}{1-s} < \dfrac{p_1}{1-p_2}$ (Using 4)

$$\Rightarrow 1+\frac{p_0}{1-p_0} < 1+\frac{s}{1-s} < 1+\frac{p_1}{1-p_1} \Rightarrow \frac{1}{1-p_0} < \frac{1}{1-s} < \frac{1}{1-p_1} \text{ or } 1-p_1 < 1-s < 1-p_0 \text{ or } p_0 < s < p_1.$$

Example 1·18. *It is desired to run a risk of 1 in 100 in rejecting a lot which is as good as 15% defective and 2 in accepting a lot which is as bad as 30% defective. Draw the decision lines and plot the OC and ASN curves for the above sequential sampling plan. How many units would you require to arrive at a decision for the following sequence of inspected items :*
"N N D N N N N N N N N N N N N N D
N N N N N N N N N D NNN, NNN, NN".
where D stands for defective item and N for non-defective item.

Solution. For the above sequential sampling plan, we have in the usual notations :
p_0 (AQL) = 0·15; p_1 (LTFD) = 0·30 ; α ((Producer's risk) = 0·01 ; β(Consumer's risk) = 0·02

$$a = \log A = \log\left(\frac{1-\beta}{\alpha}\right) = \log\left(\frac{0{\cdot}98}{0{\cdot}01}\right) = 1{\cdot}99123$$

$$b = -\log B = \log\left(\frac{1}{B}\right) = \log\left(\frac{1-\alpha}{\beta}\right) = \log\left(\frac{\cdot 99}{\cdot 02}\right) = 1{\cdot}69461$$

$$\log\left(\frac{p_1}{p_0}\right) = \log\left(\frac{0{\cdot}30}{0{\cdot}15}\right) = 0{\cdot}30103$$

$$\log\left(\frac{1-p_1}{1-p_0}\right) = \log\left(\frac{0{\cdot}70}{0{\cdot}85}\right) = \log 0{\cdot}8235 = \bar{1}{\cdot}91567$$

$$g_1 = \log\frac{p_1}{p_0} = 0{\cdot}30103$$

$$g_2 = \log\left(\frac{1-p_0}{1-p_1}\right) = -\log\left(\frac{1-p_1}{1-p_0}\right)$$

$$= -(\bar{1}{\cdot}91567) = 0{\cdot}08433$$

$$g_1 + g_2 = 0{\cdot}30103 + 0{\cdot}08433 = 0{\cdot}38536$$

$$s = \frac{g_2}{g_1+g_2} = \frac{0{\cdot}08433}{0{\cdot}38536} = 0{\cdot}2188$$

$$h_1 = \frac{b}{g_1+g_2} = \frac{1{\cdot}69461}{0{\cdot}38536} = 4{\cdot}3975$$

$$h_2 = \frac{a}{g_1+g_2} = \frac{1{\cdot}99123}{0{\cdot}38536} = 5{\cdot}1672$$

Hence, the acceptance and rejection lines are given by : [*c.f.* (1·24*e*) and *c.f.* (1·24*g*)]

Acceptance Line (L_1)

$$d_m = -h_1 + sm$$

$$\Rightarrow \quad d_m = -4{\cdot}3975 + 0{\cdot}2188m \qquad \ldots(*)$$

Rejection Line (L_2).

$$d_m = h_2 + sm$$

$$\Rightarrow \quad d_m = 5{\cdot}1672 + 0{\cdot}2188m \qquad \ldots(**)$$

For plotting the lines in (*) and (**), we need two points for each line which are obtained in the following table :

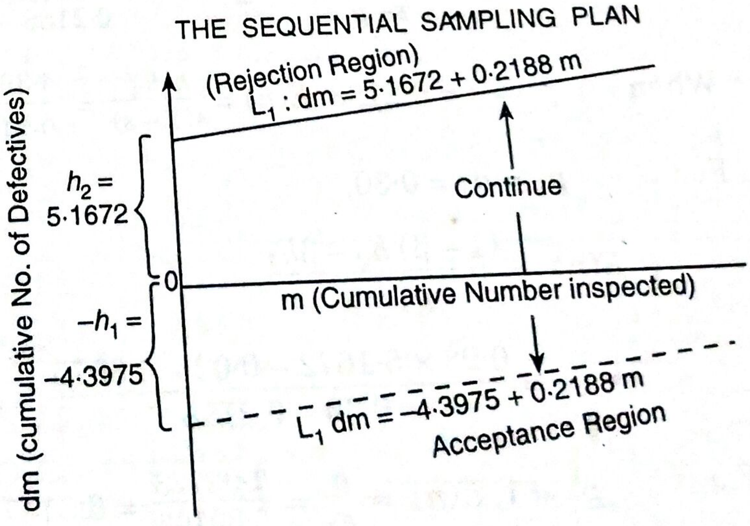

Fig. 1·25

$$\therefore \quad E(n) = \frac{a}{g_1} = \frac{h_2}{1-s} \qquad \ldots(1\cdot33e)$$

When $\quad p = s, L(p) = \dfrac{h_2}{h_1 + h_2}$ [c.f. (1·30b) and (1·30c)]

$$\therefore \quad E(n) = \frac{-\left(\frac{h_2}{h_1+h_2}\right) b + \left(\frac{h_1}{h_1+h_2}\right) a}{sg_1 - (1-s)\, g_2}$$

$$= \frac{ah_1 - bh_2}{h_1 + h_2} \; \frac{1}{s(g_1+g_2) - g_2} \left(\frac{0}{0} \text{ Form}\right)$$

[From (1·24i) and (1·30b)]

$$= \frac{h_1 h_2}{s\,(1-s)} \qquad \ldots(1\cdot33\,f)$$

These five points obtained in equations (1·33) to (1·33f) are expressed in the tabular form in the adjoining table :

FIVE POINTS ON ASN CURVE FOR SEQUENTIAL SAMPLING PLAN

P	ASN
0	$\dfrac{b}{g_2}$ or $\dfrac{h_1}{s}$
p_0 (AQL)	$\dfrac{\alpha\,(a+b) - b}{p_0\,(g_1+g_2) - g_2}$ or $\dfrac{(1-\alpha)\, h_1 - \alpha h_2}{s - p_0}$
s	$\dfrac{h_1 h_2}{s(1-s)}$
p_1 (LTFD)	$\dfrac{a - \beta(a+b)}{p_1\,(g_1+g_2) - g_2}$ or $\dfrac{(1-\beta)\, h_2 - \beta h_1}{p_1 - s}$
1	$\dfrac{a}{g_1}$ or $\dfrac{h_2}{1-s}$

Remarks **1.** Although sequential inspection plan provides for an infinite number of stages, it has been established mathematically that sequential process ultimately terminals with probability one. For a detailed discussion on SPRT the reader is referred to the book 'Sequential Analysis' by A. Wald, published by John Wiley & Sons, New York (1947).

2. The chief advantage of sequential plan is the reduction in the A.S.N. As compared with single sampling plan, sequential plan requires, on the average, 33% to 50% less inspection for the same degree of protection, *i.e.*, for same values of α and β.

3. *ASN* is maximum at $p = s$.

4. $p_0 < s < p_1$

Proof. In the usual notations : $s = \dfrac{g_2}{g_1 + g_2} \quad \Rightarrow \quad 1 - s = \dfrac{g_2}{g_1 + g_2}$

Dividing, we get
$$\frac{s}{1-s} = \frac{g_2}{g_1} \qquad \ldots(1)$$

We have
$$g_1 = \log\,(p_1/p_0) = \log p_1 - \log p_0$$

Using mean value theorem from differential calculus, *viz.*,
$$f\,(b\,) - f\,(a) = (b-a)\, f\,'(c); \qquad a < c < b,$$

with $\quad f(x) = \log x$, we get

$$g_1 = (p_1 - p_0)\left(\frac{1}{c_1}\right); \qquad p_0 < c_1 < p_1 \qquad \ldots(2)$$

Similarly,
$$g_2 = \log\left(\frac{1-p_0}{1-p_1}\right) = \log\,(1-p_0) - \log\,(1-p_1)$$

$$= (p_1 - p_0)\frac{1}{c_2},\; 1 - p_1 < c_2 < 1 - p_0 \qquad \ldots(3)$$

Substituting in (1), we get
$$\frac{s}{1-s} = \frac{c_1}{c_2} \qquad \ldots(4)$$

ASN Function of Sequential Sampling Plan. The sample size n in sequential testing is a random variable which can be determined in terms of the density function $f(x, \theta)$. The ASN function of an SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ is given by :

$$E(n) = \frac{L(\theta) \log L(\theta) \log B + [1 - L(\theta)] \log A}{E(z)}, \text{ where } z = \log \frac{f(x, \theta_1)}{f(x, \theta_0)} \quad \ldots(1\cdot31)$$

Thus for sequential sampling plan with AQL p_0 and LTFD p_1 (*i.e.*, for testing $H_0 : p = p_0$ against $H_1 : p = p_1$), we have

$$E(n) = \frac{L(p) \log B + [1 - L(p)] \log A}{E(z)} \quad \ldots(1\cdot32)$$

where $$z = \log \frac{f(x, p_1)}{f(x, p_0)} \; ; \; A = \frac{1-\beta}{\alpha} \; ; \; B = \frac{\beta}{1-\alpha} \quad \ldots(1\cdot32a)$$

$$E(z) = E \log \frac{f(x, p_1)}{f(x, p_0)} = \sum_{x=0}^{1} f(x, p). \log \frac{f(x, p_1)}{f(x, p_0)}$$

$$= p \log \frac{p_1}{p_0} + (1-p) \log \frac{1-p_1}{1-p_0}$$

Hence, $$E(n) = \frac{L(p) \log B + [1 - L(p)] \log A}{p \log \left(\frac{p_1}{p_0}\right) + (1-p) \log \left(\frac{1-p_1}{1-p_0}\right)} \quad \ldots(1\cdot32b)$$

which is the required ASN function.

Five Points on ASN Curve. A sufficiently good idea of ASN curve for the sequential sampling plan can be obtained from suitably chosen five points which are easy to obtain. The ASN curve so obtained is referred to as 5-point ASN curve.

The general 5-points on ASN curve corresponding to $p = 0, 1, p_1$ (LTFD), p_0 (AQL) and s are obtained from (1·32b) as explained below.

When $p = 0, L(p) = 1$

$$\therefore \quad E(n) = \frac{\log B}{\log [(1-p_1)/(1-p_2)]} = \frac{-b}{-g_2} = \frac{b}{g_2} = \frac{b/(g_1 + g_2)}{g_2/(g_1+g_2)} = \frac{h_1}{s} \quad \ldots(1\cdot33)$$

When $p_1 = p, L(p_1) = \beta$

$$\therefore \quad E(n) = \frac{\beta \log B + (1-\beta) \log A}{p_1 \log \frac{p_1}{p_2} + (1-p_1) \log \frac{1-p_1}{1-p_0}}$$

$$= \frac{a - (a+b)\beta}{p_1 g_1 - (1-p_1) g_2} = \frac{(1-\beta)a - b\beta}{(g_1+g_2) - g_2} \quad \ldots(1\cdot33a)$$

$$= \frac{(1-\beta) h_2 - \beta h_1}{p_1 - s} \text{ [Dividing numerator and denominator by } g_1 + g_2] \quad \ldots(1\cdot33b)$$

When $p = p_0, L(p_0) = 1 - \alpha$

$$\therefore \quad E(n) = \frac{-(1-\alpha) b + \alpha a}{p_0 g_1 - (1-p_0) g_2} = \frac{\alpha(a+b) - b}{p_0 (g_1+g_2) - g_2} \quad \ldots(1\cdot33c)$$

$$= \frac{-(1-\alpha) h_1 + \alpha h_2}{p_0 - s}$$

$$= \frac{(1-\alpha) h_1 - \alpha h_2}{s - p_0} \quad \ldots(1\cdot33d)$$

When $p = 1, L(p) = 0$

Remark. If h assumes negative values, *i.e.*, if instead of h we take $-h$ where now $h > 0$, then

$$L(p, -h) = \frac{A^{-h}-1}{A^{-h}-B^{-h}} = \left(\frac{1-A^h}{B^h-A^h}\right)B^h = \left(\frac{A^h-1}{A^h-B^h}\right)B^h \quad \text{...(1·29)}$$

$$\therefore \quad L(p, -h) = B^h \,.\, L(p, h)$$

and

$$p(-h) = \frac{\left(\frac{1-p_1}{1-p_0}\right)^h - 1}{\left(\frac{1-p_1}{1-p_0}\right)^h - \left(\frac{p_1}{p_0}\right)^h} \cdot \left(\frac{p_1}{p_0}\right)^h = p(h) \,.\, \left(\frac{p_1}{p_0}\right)^h \quad \text{...(1·29a)}$$

Thus for negative values of h, the points on the OC curve can be obtained from equations (1·29) and (1·29a).

Five Points on OC Curve. Often, a sufficient appraisal of the OC can be obtained from the following five easily computed points on the curve.

Since a lot containing no defective ($p = 0$) will always be accepted and a lot with 100% defective ($p = 1$) is sure to be rejected, we have

$$L(0) = 1 \quad \text{and} \quad L(1) = 0$$

$$\begin{aligned} L(p_0) &= P\,(\text{Accepting a lot of quality } p_0) \\ &= 1 - P\,(\text{Rejecting a lot of quality } p_1) \\ &= 1 - \alpha \\ L(p_1) &= P\,(\text{Accepting a lot of quality } p_1) = \beta \end{aligned}$$

Let $p = p'$ when $h = 0$, *i.e.*, $p' = \lim\limits_{h_0 \to 0} p = \lim\limits_{h_0 \to 0} \dfrac{1 - \left(\frac{1-p_1}{1-p_0}\right)^h}{\left(\frac{p_1}{p_0}\right)^h - \left(\frac{1-p_1}{1-p_0}\right)^h}$ ...(1·30)

This is the indeterminate form $\frac{0}{0}$ and hence by L'Hospital's rule, we get

$$p' = \lim_{h \to 0} \frac{\left(\frac{1-p_1}{1-p_0}\right)^h \log\left(\frac{1-p_1}{1-p_0}\right)}{\left(\frac{p_1}{p_0}\right)^h \log\frac{p_1}{p_0} - \left(\frac{1-p_1}{1-p_0}\right)^h \log\frac{1-p_1}{1-p_0}} = \frac{-\log\left(\frac{1-p_1}{1-p_0}\right)}{\log\left(\frac{p_1}{p_0} - \log\frac{1-p_1}{1-p_0}\right)}$$

$$p' = \lim_{h \to 0} L(p) = \lim_{h \to 0} \frac{A^h - 1}{A^h - B^h} = \frac{\log A}{\log A - \log B} \quad \text{(L'Hospital's Rule)} \quad \text{...(1·30a)}$$

Remark. Using the notations of (1·24d), we get from (1·30) and (1·30a)

$$p' = \frac{g_2}{g_1 + g_2} = s \quad \text{...(1·30b)}$$

$$L(p') = \frac{a}{a+b} = \frac{h_2}{h_1 + h_2} \quad \text{...(1·30c)}$$

[On dividing numerator and denominator by $g_1 + g_2$ and using (1·24f) and (1·24h).]

The five points for the OC curve are expressed in a tabular form in the adjoining table :

h	p	$L(p)$
∞	0	1
1	$p_0 = AQL$	$1 - \alpha$
0	p'	$\frac{a}{a+b} = \frac{h_2}{h_1+h_2}$
-1	$p_1 = LTFD$	β
$-\infty$	1	0

It is obvious from the equations (1·24*e*) and (1·24*g*) that the acceptance and the rejection lines are parallel to each other, their slope being s.

It may be pointed out that d_m is the cumulative number of defectives, m is the cumulative number of observations, at the stage considered.

First of all, we plot the two lines L_1 and L_2. If at any stage the point (m, d_m) lies between the two lines, the sampling is to be continued by taking an additional observation. If the point (m, d_m) lies above or one line L_2, the lot is rejected and if the point (m, d_m) lies below or on the line L_1, lot is accepted.

Remark. Dividing (1·24 *f*) by (1·24*h*), we get :

$$\frac{h_1}{h_2} = \frac{b}{a} \quad \Rightarrow \quad ah_1 - bh_2 = 0 \qquad \text{...(1·24 } i\text{)}$$

OC of Sequential Sampling Plan. The *OC* function of a SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ in sampling from population with density function $f(x, \theta)$ is given by :

$$L(\theta) = P_a(\theta) = \frac{A^h(\theta) - 1}{A^h(\theta) - B^h(\theta)} \qquad \text{...(1·25)}$$

where, for each value of θ, the value of $h(\theta)$ is to be determined so that $h(\theta) \neq 0$ and

$$E\left[\frac{f(x, \theta_1)}{f(x, \theta_0)}\right]^{h(\theta)} = 1, \qquad \text{...(1·25}a\text{)}$$

where A and B have been defined in (1·23*b*).

Thus the *O.C.* function of S.P.R.T. for testing $H_0 : p = p_0$ against $H_1 : p = p_1$ is given by :

$$L(p) = \frac{A^h - 1}{A^h - B^h} = L(p, h), \text{ (say)} \qquad \text{...(1·26)}$$

where $h = h(p)$ is obtained by the equation :

$$\sum_{x=0}^{1} \left[\frac{f(x, p_1)}{f(x, p_0)}\right]^h f(x, p) = 1$$

$$\Rightarrow \quad \left[\frac{f(1, p_1)}{f(1, p_0)}\right]^h f(1, p) + \left[\frac{f(0, p_1)}{f(0, p_0)}\right]^h f(0, p) = 1$$

$$\Rightarrow \quad p\left(\frac{p_1}{p_0}\right)^h + (1-p)\left(\frac{1-p_1}{1-p_0}\right)^h = 1 \qquad \text{...(1·27)}$$

The solution of (1·27) for $h = h(p)$ is very tedious. From practical. point of view, to draw the *OC* curve, it is necessary to solve (1·27) for *hg* instead we may regard h as a parameter and solve (1·27) for p thus giving

$$p = \frac{1 - \left(\dfrac{1-p_1}{1-p_0}\right)^h}{\left(\dfrac{p_1}{p_0}\right)^h - \left(\dfrac{1-p_1}{1-p_0}\right)^h} = p(h), \text{ (say)} \qquad \text{...(1·28)}$$

Now, various points on the *OC* curve are obtained by giving arbitrary values to h and computing corresponding values of p and $L(p)$ from (1·28) and (1·26) respectively.

$$\log \lambda_m = d_m \log\left(\frac{p_1}{p_0}\right) + (m - d_m)\log\left(\frac{1-p_1}{1-p_0}\right)$$

Hence *accept the lot if*

$$d_m \log\left(\frac{p_1}{p_0}\right) + (m - d_m)\log\left(\frac{1-p_1}{1-p_0}\right) \le \log B \quad \Rightarrow d_m \le \frac{\log B - m\log\left(\frac{1-p_1}{1-p_0}\right)}{\log\left(\frac{p_1}{p_0}\right) - \log\left(\frac{1-p_1}{1-p_0}\right)} = a_m \text{ (say)} \quad \text{...(1·24a)}$$

Reject the lot if

$$d_m \log\left(\frac{p_1}{p_0}\right) + (m - d_m)\log\left(\frac{1-p_1}{1-p_0}\right) \ge \log A \Rightarrow d_m \ge \frac{\log A - m\log\left(\frac{1-p_1}{1-p_0}\right)}{\log\left(\frac{p_1}{p_0}\right) - \log\left(\frac{1-p_1}{1-p_0}\right)} = r_m \text{ (say)} \quad \text{...(1·24b)}$$

Continue sampling if

$$a_m < d_m < r_m \quad \text{...(1·24c)}$$

For each m, a_m and r_m are known as acceptance number and rejection number respectively.

Procedure. At each stage of the experiment, we compute a_m and r_m and we continue inspection as long as $a_m < d_m < r_m$. The first time when this inequality is violated, the inspection is stopped and then

(*i*) if $d_m \ge r_m$, lot is rejected, and

(*ii*) if $d_m \le a_m$, lot is accepted.

Remark. If we write

$$\left.\begin{aligned} g_1 &= \log (p_1/p_0),\ g_2 = \log\left(\frac{1-p_0}{1-p_1}\right);\ \log A = a,\ \log B = -b \\ \text{and} \quad s &= \frac{\log (1-p_0/1-p_1)}{\log (p_1/p_0) - \log (1-p_1/1-p_0)} = \frac{g_2}{g_1+g_2} \end{aligned}\right\} \quad \text{...(1·24d)}$$

then the acceptance and rejection lines L_1 and L_2 are given by the following equations :

Acceptance Line L_1 :

$$d_m = a_m = \frac{-b}{g_1+g_2} + \frac{mg_2}{g_1+g} \quad \Rightarrow \quad d_m = -h_1 + sm \quad \text{...(1·24e)}$$

where

$$h_1 = \frac{b}{g_1+g_2} \quad \text{...(1·24f)}$$

and $-h_1$ gives the intercept of the line L_1 on the d_m axis.

Rejection Line L_2 :

$$d_m = r_m = \frac{a}{g_1+g_2} + m\frac{g_2}{g_1+g_2} \quad \Rightarrow \quad d_m = h_2 + sm \quad \text{...(1·24g)}$$

where $h_2 = \dfrac{a}{g_1+g_2}$ ...(1·24h)

is the intercept of the line L_2 on the d_m axis.

The ultimate in multiple sampling is sequential sampling which provides for infinite number of stages for arriving at a decision. In sequential sampling, sample items are examined one at a time and after each item inspected one of three decisions, *viz.*, to accept the lot, to reject the lot or to continue sampling is taken. This scheme provides for a minimum amount of inspection.

Sequential schemes are considered to require most care and supervision in operation. Where the inspection or testing costs per article are high and sampling destructive, utmost economy in the number of articles inspected is important and often outweighs administrative convenience.

Sequential Probability Ratio Test **(S.P.R.T.).** A sampling plan satisfying the condition that the probability of rejecting the lot does not exceed α whenever $p \le p_0$ and the probability of accepting lots does not exceed β whenever $p \ge p_1$ is given by the *sequential probability ratio test* (SPRT), pioneered by Dr. Abraham Wald, for testing the hypothesis H_0 : $p = p_0$ against the hypothesis $H_1 : p = p_1$.

Here if we take AQL = p_0 ; LTPD = $100p_1$ or lot tolerance fraction defective p_1 : α = Probability of Type I error and β = Probability of Type II error then α and β are the maximum producer's and consumer's risks respectively. SPRT is defined as follows :

Let the result of the inspection of the ith unit be denoted by a Bernoulli variate X_i , *i.e.*,

$$X_i = 1, \text{ if } i\text{th item inspected is found to be defective}$$
$$= 0, \text{ otherwise.}$$

For the incoming lot quality 'p', if $f(x, p)$ represents the probability function of X then

$$f(1, p) = p \quad \text{and} \quad f(0, p) = 1 - p$$

Let p_{1m} and p_{0m} be the probabilities of getting d_m defectives in the sample $(X_1, X_2, ..., X_m)$ of size m under H_1 and H_0 respectively. Then the Likelihood Ratio λ_m is given by :

$$\lambda_m = \frac{p_{1m}}{p_{om}} = \frac{\prod_{i=1}^{m} f(x_i, p_1)}{\prod_{i=1}^{m} f(x_i, p_0)} = \prod_{i=1}^{m} \frac{f(x_i, p_1)}{f(x_i, p_0)} = \frac{p_1^{dm} (1 - p_1)^{m - d_m}}{p_0^{dm} (1 - p_0)^{m - d_m}} \quad ...(1·23)$$

SPRT *is carried out as follows* : At each stage of the experiment, at the inspection of the mth for each possible integral value m, we compute λ_m and

(*i*) If $\lambda_m \ge A$, we terminate the process with rejection of the lot.
(*ii*) If $\lambda_m \le B$, we terminate the process with acceptance of the lot.
(*iii*) If $B < \lambda_m < A$, we continue the sampling by taking an additional observation, ...(1·23*a*)

where A and B are constants determined in terms of α and β and are given by

$$A = (1 - \beta)/\alpha \quad \text{and} \quad B = \beta/(1 - \alpha) \quad ...(1·23b$$

For computational points of view, it would be much easier to deal with log λ_m rather tha with λ_m. Thus SPRT can be restated as follows :

(*i*) If $\lambda_m \ge \log \log A$, reject the lot,
(*ii*) If $\log \lambda_m \le \log B$, accept the lot, and
(*iii*) If $\log B < \log \lambda_m < \log A$, continue sampling by taking one more observation. ...(1·23

Since $P_a = P_{a_1} + P_{a_2} \Rightarrow P_{a_2} = P_a - P_{a_1}$,

we get from (1·22)

$$ATI = n_1 P_{a_1} + (n_1 + n_2)(P_a - P_{a_1}) + N(1 - P_a)$$
$$= n_1 P_{a_1} + (n_1 + n_2)[(1 - P_{a_1}) - (1 - P_a)] + N(1 - P_a)$$
$$= n_1 + n_2(1 - P_{a_1}) + (N - n_1 - n_2)(1 - P_a) \quad \ldots(1·22a)$$

Remark. In Dodge and Romig tables, n_2 has no fixed relation to n_1 but is determined so that ATI is minimum and so that the probability of acceptance on the basis of first sample is approximately the same as the probability of acceptance on the basis of second sample.

1·12·3. Single Sampling *vs.* Double Sampling Plans

1. Single sampling plans are simple, easy to design and administer, and since each sample can be plotted on a control chart, maximum information concerning the lot can be obtained.

2. A very important advantage of double sampling over single sampling seems to be psychological. To a layman, it seems unfair to reject a lot on the basis of one sample alone and appears more convincing to say that the lot was rejected after inspecting two samples. Moreover, in double sampling no lot can be rejected without finding at least two defectives in the sample taken from it—thus the border-line lots (lots of marginal quality) always get a second chance of being accepted.

3. In the conditions under which the sampling schemes are generally operated it has been found that the double sampling scheme involves on the average less amount of inspection than the single sampling scheme for the same quality assurance. Under the double sampling scheme the good quality lot will generally be accepted and bad lots will usually be rejected on the basis of the first sample. Thus, in all the cases, where a decision to accept or reject is taken on the basis of the first sample, there is a considerable saving in the amount of inspection than required by a comparable (*w.r.t.* OC curve) single sampling plan. Moreover, whenever a second sample is taken it may be possible to reject the lot without completely inspecting the entire second sample. Usually double sampling requires 25% to 33% less inspection on the average, than single sampling.

The general reduction in the amount of inspection afforded by double sampling is one of its strongest advantages. This does not necessarily mean, however, that a double sampling scheme could be less costlier than the single sampling scheme. The double sampling schemes being more complicated and the necessity of inspecting second sample being unpredictable, the unit cost of inspection for a double sampling procedure may be higher than that for single sampling procedure.

4. The operating characteristic curves of double sampling scheme are generally steeper than those of corresponding single sampling procedure *i.e.*, the discriminatory power of double sampling procedures is a bit higher than that of single sampling procedures.

1·12·4. Sequential Sampling Plan. We know that one of the advantages of double sampling over single sampling is psychological in giving the lot second chance for acceptance. Moreover, except for lots of marginal quality the average amount of inspection in double sampling is less for the same protection. It is, therefore, natural to suggest triple, quadruple or in general multiple sampling as a way to reduce the amount of inspection still further. Unfortunately such plans become very complex both to construct and to administer and the small gain in sampling reduction is inefficient to warrant them unless one goes all the way to *sequential sampling*.

The ultimate in multiple sampling is sequential sampling which provides for infinite number of stages for arriving at a decision. In sequential sampling, sample items are examined one at a time and after each item inspected one of three decisions, *viz.*, to accept the lot, to reject the lot or to continue sampling is taken. This scheme provides for a minimum amount of inspection.

Sequential schemes are considered to require most care and supervision in operation. Where the inspection or testing costs per article are high and sampling destructive, utmost economy in the number of articles inspected is important and often outweighs administrative convenience.

Sequential Probability Ratio Test **(S.P.R.T.)**. A sampling plan satisfying the condition that the probability of rejecting the lot does not exceed α whenever $p \le p_0$ and the probability of accepting lots does not exceed β whenever $p \ge p_1$ is given by the *sequential probability ratio test* (SPRT), pioneered by Dr. Abraham Wald, for testing the hypothesis H_0 : $p = p_0$ against the hypothesis $H_1 : p = p_1$.

Here if we take AQL = p_0 ; LTPD = $100p_1$ or lot tolerance fraction defective p_1 : α = Probability of Type I error and β = Probability of Type II error then α and β are the maximum producer's and consumer's risks respectively. SPRT is defined as follows :

Let the result of the inspection of the *i*th unit be denoted by a Bernoulli variate X_i , *i.e.*,

$$X_i = 1, \text{ if } i\text{th item inspected is found to be defective}$$
$$= 0, \text{ otherwise.}$$

For the incoming lot quality 'p', if $f(x, p)$ represents the probability function of X then

$$f(1, p) = p \quad \text{and} \quad f(0, p) = 1 - p$$

Let p_{1m} and p_{0m} be the probabilities of getting d_m defectives in the sample $(X_1, X_2, ..., X_m)$ of size m under H_1 and H_0 respectively. Then the Likelihood Ratio λ_m is given by :

$$\lambda_m = \frac{p_{1m}}{p_{om}} = \frac{\prod_{i=1}^{m} f(x_i, p_1)}{\prod_{i=1}^{m} f(x_i, p_0)} = \prod_{i=1}^{m} \frac{f(x_i, p_1)}{f(x_i, p_0)} = \frac{p_1^{d_m} (1 - p_1)^{m - d_m}}{p_0^{d_m} (1 - p_0)^{m - d_m}} \quad ...(1·23)$$

SPRT *is carried out as follows* : At each stage of the experiment, at the inspection of the *m*th for each possible integral value m, we compute λ_m and

(*i*) If $\lambda_m \ge A$, we terminate the process with rejection of the lot.
(*ii*) If $\lambda_m \le B$, we terminate the process with acceptance of the lot.
(*iii*) If $B < \lambda_m < A$, we continue the sampling by taking an additional observation, ...(1·23*a*)

where A and B are constants determined in terms of α and β and are given by

$$A = (1 - \beta)/\alpha \quad \text{and} \quad B = \beta/(1 - \alpha) \quad ...(1·23b)$$

For computational points of view, it would be much easier to deal with log λ_m rather than with λ_m. Thus SPRT can be restated as follows :

(*i*) If $\lambda_m \ge \log \log A$, reject the lot,
(*ii*) If $\log \lambda_m \le \log B$, accept the lot, and
(*iii*) If $\log B < \log \lambda_m < \log A$, continue sampling by taking one more observation. ...(1·23c)

Since $P_a = P_{a_1} + P_{a_2} \Rightarrow P_{a_2} = P_a - P_{a_1}$,

we get from (1·22)

$$ATI = n_1 P_{a_1} + (n_1 + n_2)(P_a - P_{a_1}) + N(1 - P_a)$$
$$= n_1 P_{a_1} + (n_1 + n_2)[(1 - P_{a_1}) - (1 - P_a)] + N(1 - P_a)$$
$$= n_1 + n_2(1 - P_{a_1}) + (N - n_1 - n_2)(1 - P_a) \quad \ldots(1·22a)$$

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